Reflection, radiation and interference for black holes

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Black holes are capable of reflection: there is a finite probability for any particle that approaches the event horizon to bounce back. The albedo of the black hole depends on its temperature and the energy of the incoming particle. The reflection shares its physical origins with the Hawking process of radiation, both of them arise as consequences of the mixing of the incoming and outgoing waves that takes place on the event horizon.

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I. INTRODUCTION

Conventional intuitive arguments attribute two defining properties to black holes. The black holes are presumed to be unable to emit anything into the outside world, and are supposed to possess perfect absorption ability, i.e. be able to take in everything that comes close to their event horizon. There is, however, a known limitation for this simple intuitive picture that stems from thermodynamics that attributes temperature and entropy to black holes. The first indication that gravitational fields could have entropy came when investigation of Christodoulou [1] of the Penrose process [2] for extracting energy from a Kerr black hole showed that there is a quantity which could not go down. Hawking found [3] that it is proportional to the area of the horizon. Further research of Bardeen *et al* [4] demonstrated that black holes should obey laws similar to the laws of thermodynamics. An important step made by Bekenstein [5, 6, 7] revealed that the area was actually the physical entropy. This suggestion was supported and enriched by the discovery of the Hawking radiation phenomenon [8, 9]. These works provided foundation for the thermodynamics approach to the black holes, for a recent review see Wald [10] and references therein, see also books of Frolov and Novikov [11], Thorne [12], and Chandrasekhar [13] for comprehensive discussion of other black hole properties.

The thermodynamics properties of black holes reveal that a black hole has the finite temperature T and, correspondingly, is capable of radiation through the Hawking mechanism, in contradiction with the naive expectations. In this work we address another property of black holes, their ability for an absorption. From the first glance the Hawking mechanism of radiation supports an ability of black holes for perfect absorption. Indeed, the radiation spectrum of a black hole coincides with the spectrum of the perfect black body. It is natural therefore to presume that the black hole is the black body, and as such should be a ideal absorber. However, we show that this perception is inadequate. The black holes are capable to reflect particles that come to their event horizon, and henceforth the black hole should not be considered as the perfect black body. The fact that the radiation spectrum of the black hole coincides with the black body spectrum produces no contradiction. Interestingly, the radiation properties of the black hole and its reflection ability have one and the same physical origin.

The classical description of the motion in the vicinity of the black hole horizon includes two types of trajectories. There are the ingoing trajectories, they describe the motion towards the black hole center. There are also the outgoing trajectories that lead out of the black hole center. Classically these two types of motion are quite different. If a particle, following the ingoing trajectory, approaches the event horizon, then it inevitably crosses it into the inside region. After that it stays inside, there is no classically allowed way for it to switch to any outgoing trajectory that leads into the outside region, in full accord with intuitive feelings. Discussing this point later we will use Fig. 1 as an illustration to this statement. The quantum description reveals a new rather unexpected feature of the problem. The event horizon produces a strong impact on the wave function of a probing particle. The effect can be described in terms of interference or, equivalently, mixing of the incoming and outgoing waves. The incoming wave corresponds to the incoming classical trajectory; the correspondence between the waves and the trajectories works well in the vicinity of the event horizon because here the semiclassical description is applicable. The trajectory describes a smooth crossing of the horizon. Similarly, the outgoing trajectory also describes the smooth transition through the horizon. However the quantum description reveals that some events that happen strictly on the horizon produce strong impact on the wave function, mixing the incoming wave with the outgoing one. In other words, the wave function of the incoming particle necessarily includes both the incoming and outgoing waves.

The presence of the outgoing wave in the wave function has important physical implications. One of them is the effect of reflection that is discussed in some detail in this work. The reflection means that there is a finite probability for an incoming particle to be reflected off the

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event horizon back, to the outside region. Another effect is the well known Hawking mechanism of radiation. It is demonstrated that the radiation can be considered as a consequence of the mentioned interference. This new point of view provides an attractive physical picture that makes more clear some details on the radiation process

To be more specific consider a particle in the outside region that approaches the black hole horizon. It is shown that there is a finite probability $\mathcal{P}$ for the particle to be reflected off the horizon,

$$\mathcal{P} = \exp\left(-\frac{\varepsilon - Q\Phi - J\omega}{kT}\right) . \tag{1.1}$$

This probability depends on the energy of the incoming particle ε, its charge Q, and its projection of the orbital momentum J on the axis of rotation of the black hole. The essential parameters of the black hole that govern the process are the temperature T, the electric potential on the horizon Φ, and the angular velocity of the horizon ω. The lower is the temperature, the stronger is the ability for reflection.

Notably, the probability of reflection (1.1) coincides with the temperature factor that governs the Hawking radiation process, though the physical manifestation of the reflection differs from the radiation since the flux of the reflected particles is proportional to the magnitude of the incoming flux. Nevertheless, a similarity between the probability of reflection (1.1) and the temperature factor is not accidental. As was mentioned above, the reflection and radiation share the same physical origin, namely the interference of the incoming and outgoing waves. A convenient way to prove the existence of this interference and to examine its magnitude is presented. Eq.(1.1) is derived as a consequence of the interference for the reflection process, and a new derivation for the radiation process is discussed.

Relativistic units $\hbar = c = 1$ supplemented by the condition $2Gm = 1$ imposed on the gravitational constant G and the black hole mass m are used, if not stated otherwise. The Schwarzschild radius in these units reads simply $r_g = 2Gm/c^2 \equiv 1$.

II. SINGULARITY OF THE WAVE FUNCTION ON THE HORIZON

Consider the static black hole described by the conventional Schwarzschild metric

$$ds^2 = -\left(1 - \frac{1}{r}\right) dt^2 + \frac{dr^2}{1 - 1/r} + r^2 d\Omega^2 , \tag{2.1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$. The Hamilton-Jacobi classical equations of motion $g^{\kappa\lambda}\partial_\kappa S \partial_\lambda S = -\mu^2$ for a particle with the mass μ in the metric (2.1) take the form

$$\frac{\dot{S}^2}{1 - 1/r} = \left(1 - \frac{1}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 + \mu^2 . \tag{2.2}$$

Separating the variables $S(\mathbf{r}, t) = -\varepsilon t + L\varphi + S(r)$ where ε and L are the energy and the momentum of the particle, φ is its azimuthal angle, one finds the radial action

$$S(r) = \mp \int^r \left[\varepsilon^2 - \left(\mu^2 + \frac{L^2}{r}\right)\left(1 - \frac{1}{r}\right)\right]^{1/2} \frac{dr}{1 - 1/r} . \tag{2.3}$$

In the vicinity of the black hole horizon $r \to 1$, which plays an important role in the following discussion, the action (2.3) simplifies

$$S(r) = \mp\, \varepsilon \ln(r - 1) , \tag{2.4}$$

that gives $S(\mathbf{r}, t) = -\varepsilon t \pm \varepsilon \ln(r - 1) + L\varphi$. The corresponding equation of motion $\partial_\varepsilon S(\mathbf{r}, t) = 0$ yields the radial trajectories

$$r = 1 + \exp(\mp t) . \tag{2.5}$$

The signs minus and plus in Eqs.(2.3),(2.4),(2.5) correspond to the incoming and outgoing trajectories respectively. These equations are conveniently written for the outside region $r > 1$ (the inside region is discussed in Section V). It is important that the classical action for a probing has the logarithmic singularity (2.4) on the horizon. The coefficient in front of the logarithm function is equal to the energy of the particle ε ($\varepsilon r_g/c$ in absolute units) that plays an important role in what follows, eventually finding its way into the exponential function in (1.1). Importantly, the logarithmic singularity is an invariant property of the action, it persists even in those coordinates that eliminate the singularity of the metric on the horizon. For example, in Kruskal coordinates U,V [14] (for a comprehensive discussion of the Kruskal coordinates see Ref.[15])

$$U = -\ \sqrt{r - 1}\ \exp[\,(r - t)/2\,] , \tag{2.6}$$

$$V = \ \sqrt{r - 1}\ \exp[\,(r + t)/2\,] , \tag{2.7}$$

the metric $ds^2 = -4dUdV \exp(-r)/r + r^2 d\Omega^2$ is regular on the horizon, which is described by conditions $U = 0$ or $V = 0$, but the logarithmic singularity of the action remains intact, it can be conveniently presented as

$$S(\mathbf{r}, t) \simeq -\ \varepsilon \ln(V^2) , \qquad \text{for} \quad U \to 0 , \tag{2.8}$$

$$S(\mathbf{r}, t) \simeq \ \varepsilon \ln(U^2) , \qquad \text{for} \quad V \to 0 . \tag{2.9}$$

The classical action allows one to find the semiclassical wave function $\Phi(\mathbf{r}, t)$ that describes the coordinate motion of the particle (leaving aside possible spin variables). Separating the variables, $\Phi(\mathbf{r}, t) = \exp(-i\varepsilon t) Y_{LM}(\theta, \varphi)\phi(r)$, where $Y_{LM}(\theta, \varphi)$ is the conventional spherical function describing the motion with the orbital momentum L and its projection M, one presents the semiclassical radial wave function $\phi(r)$ as

$$\phi(r) \propto \exp[\,iS(r)\,] \simeq \exp[\,\mp i\, \varepsilon \ln(r - 1)\,] . \tag{2.10}$$

It is verified below, see after (2.11), that the preexponential factor in (2.10) is a constant, which we chose to be

unity. Thus the singularity of the action at $r = 1$ results in the corresponding singularity of the wave function.

In order to scrutinize this result one needs to assess validity of the semiclassical description in the vicinity of the horizon. To this end consider the wave function $\Phi(\mathbf{r}, t)$ as a solution of the Klein-Gordon equation for the scalar field. From (2.1) one finds that the radial wave function $\phi(r)$ satisfies the equation

$$\begin{aligned} &\phi'' + \left(\frac{1}{r} + \frac{1}{r-1}\right)\phi' \\ &+ \frac{1}{1-1/r}\left(\frac{\varepsilon^2}{1-1/r} - \mu^2 - \frac{L(L+1)}{r^2}\right)\phi = 0 \ . \end{aligned} \tag{2.11}$$

In the vicinity of the horizon $r = 1$ the solution can be approximated by $\phi(r) \simeq (r-1)^\eta$ where (2.11) yields $\eta = \pm i\varepsilon$. The agreement with the semiclassical result (2.10) supports its validity and verifies that the preexponential factor in (2.10) is, indeed, a constant. It is instructive also to look at the singularity of the wave function (2.10) from the point of view of the conventional Schrödinger-type equation. Making the substitution $\phi(r) \to \psi(r) = [\, r(r-1)\,]^{1/2}\phi(r)$ one rewrites (2.11)

$$p^2\psi(r) = -\psi''(r) + U(r)\psi(r) \ , \tag{2.12}$$

where

$$\begin{aligned} U(r) = \ &- \ \frac{1}{(r-1)^2}\left(\varepsilon^2 + \frac{1}{4r^2}\right) \\ &- \ \frac{1}{r-1}\left(\varepsilon^2 + p^2 - \frac{L(L+1)}{r}\right) . \end{aligned} \tag{2.13}$$

Eq.(2.13) has the form of the Schrödinger-type equation if we consider $U(r)$ as an effective, energy-dependent potential and accept the momentum p^2 on the left-hand side as the eigenvalue. For $r \to 1$ the potential exhibits a notable feature

$$U(r) \to -\frac{\varepsilon^2 + 1/4}{(r-1)^2} \ . \tag{2.14}$$

It is well known in nonrelativistic quantum mechanics [16] that in the potential $U(z) = -U_0/z^2$ for $U_0 > 1/4$ the wave function collapses to the point $z = 0$. Since the necessary inequality is obviously satisfied in (2.14), $\varepsilon^2 + 1/4 > 1/4$, one conclude that (2.12) indicates the collapse of the wave function on the event horizon $r = 1$. This fact could be interpreted as the absorption of the particle by the black hole. Thus from the first sight the quantum description seems to agree with classical arguments based on the incoming trajectory in (2.5) that converges to the event horizon, supporting also the intuitive perception of the black hole as an ideal absorber. However, more careful discussion below exposes limitations of this point of view.

Summarizing, it is demonstrated that the wave function $\phi(r)$ has a singularity (2.10) on the event horizon.

III. REFLECTION

Consider a particle that approaches the event horizon of the black hole. Let us describe its radial motion with the help of the wave function $\phi(r)$. According to (2.10) the wave function in the vicinity of the horizon can be written as

$$\phi(r) = \exp[-i\,\varepsilon\ln(r-1)] + \mathcal{R}\exp[i\,\varepsilon\ln(r-1)] \ . \tag{3.1}$$

The first term here describes the proper incoming wave, while the second one, that presents the outgoing wave, is written in order to allow for an opportunity of the possible interference of the incoming and outgoing waves in the wave function. If this interference takes place, i. e. if $\mathcal{R} \neq 0$, then the outgoing wave in (3.1) clearly indicates that there is the probability for the incoming particle to be reflected on the horizon. The unitarity condition implies $|\mathcal{R}| \le 1$. Moreover, intuitively one would expect the reflection coefficient in (3.1) to be zero, $\mathcal{R} = 0$. Such assumption would agree with a naive perception of the black hole as a perfect absorber. However, in order to verify, approve or reject this intuitive claim (we will reject it, in fact) one needs to examine carefully what happens with the wave function on the horizon.

Straightforward discussion of events that happen strictly at $r = 1$ faces an obstacle produced by the singular nature of the wave function (3.1) at this point. Fortunately, one can avoid discussion of the events that take place strictly on the horizon r=1 using the analytical continuation of the wave function in the vicinity of this point. Consider the distance from the horizon $z = r - 1$, treating z as a complex variable. The wave function (3.1) is explicitly analytical in z, except for the power-type singularity at $z = 0$ that induces a cut emerging from this point on the complex plane z. Let us take r in the outside region of the black hole in a close vicinity of the event horizon, which means that $0 < z \ll 1$, and examine what happens with the wave function when one rotates z in the complex z-plane over an angle 2π clockwise (the anti-clockwise rotation is forbidden, see discussion after (3.5)). We can keep $|z|$ small, $|z| \ll 1$, during this rotation, thus justifying validity of the semiclassical wave function (3.1). This analytical continuation necessarily incorporates a crossing of the cut on the complex plane. Therefore after finishing this rotation and returning to a real, physical value $z > 0$, the wave function acquires a new value on its Riemann surface, let us call it $\phi^{(2\pi)}(r)$. A procedure of this type is usually referred to as a monodromy. In our case the monodromy can be read of (3.1)

$$\phi^{(2\pi)}(r) = \varrho\exp[-i\varepsilon\ln(r-1)] + \frac{\mathcal{R}}{\varrho}\exp[i\varepsilon\ln(r-1)], \tag{3.2}$$

where $\varrho = \exp(-2\pi\varepsilon)$. The analytically continued function $\phi^{(2\pi)}(r)$ satisfies the same real differential equation as the initial function $\phi(r)$. Moreover, one has to expect that the wave function satisfies the same normalization conditions as the initial wave function $\phi(r)$. This

needs that one of the coefficients in (3.2), either ϱ, or $\mathcal{R}/\varrho$ should have an absolute value equal to unity. Since $\varrho < 1$, we deduce that $|\mathcal{R}|/\varrho = 1$ thus concluding that

$$|\mathcal{R}| = \exp\left(-\frac{2\pi r_g \varepsilon}{\hbar c}\right) , \tag{3.3}$$

where the conventional units are used to make the result more transparent. We see that the reflection coefficient is nonzero. In other words, the black hole is capable of reflection, in a notable contradiction with the naive intuitive expectations. There exists a way to scrutinize (3.3). Recall again that $\phi^{(2\pi)}(r)$ satisfies the same differential equation as the wave functions $\phi(r)$ and $\phi^*(r)$. One should be able therefore to present $\phi^{(2\pi)}(r)$ as their linear combination $\phi^{(2\pi)}(r) = \alpha\phi(r) + \beta\phi^*(r)$. From (3.1) and (3.2) we derive that this linear combination really exists, having a simple form

$$\phi^{(2\pi)}(r) = \beta\phi^*(r) , \quad |\beta| = 1 . \tag{3.4}$$

Thus the physical picture presented is self-consistent. Additional support for the results discussed provide Refs.[17, 18] that suggest alternative approaches leading to Eq.(3.1). Ref.[17] argues that Eq.(3.4) expresses the fundamental symmetry of the space-time. Starting from this symmetry condition, Ref.[17] derives Eq.(3.3). Ref.[18] adopts another point of view. It relies more heavily on the dynamics of the system represented by the wave equation (2.11). This work claims that an accurate treatment of the solution of this equation in the vicinity of its three singular points, $r = 0$, $r = 1$, $r = \infty$, leads to Eq.(3.3).

From (3.3) we see that the incoming and the outgoing waves interfere in the wave function (3.1). Correspondingly, there is the reflection. The probability of reflection can be found as $\mathcal{P} = |\mathcal{R}|^2$, that in view of (3.3) gives $\mathcal{P} = \exp(-4\pi\varepsilon)$ in full accord with (1.1) proclaimed in Section I. The parameter T that appears in (1.1) arises from the coefficient in front of the logarithmic function in (2.4)

$$kT = \frac{\hbar c}{4\pi r_g} , \tag{3.5}$$

(absolute units). Notably, it proves be equal to the Hawking temperature of the black hole. Applying (1.1) one should remember, of course, that the electric potential and rotational frequency for the Schwarzschild case are absent, $\Phi = \omega = 0$.

Let us return back to examine why it was necessary to use specifically the clockwise rotation when the analytical continuation of the wave function (1.1) in the complex z-plane was fulfilled. A simplified answer to this question is that an attempt to use the counter-clockwise rotation leads to a self-contradiction. Trying it, i.e. making the anti-clockwise rotation, one arrives to the result similar to (3.2), but with the different coefficient ϱ' instead of ϱ, $\varrho \to \varrho' = 1/\varrho = \exp(2\pi\varepsilon)$. Proceeding further one would be forced to conclude that the reflection coefficient is $|\mathcal{R}'| = \exp(2\pi\varepsilon)$, which comes into an obvious contradiction with the unitarity condition for the reflection that specifies $|\mathcal{R}'| \le 1$. It is a known, common feature of the semiclassical wave function that different ways for its analytical continuation lead to different results, and one needs to choose carefully an appropriate way of continuation (3.1). To outline deeper roots of this problem in our specific case it is convenient to use Kruskal coordinates (2.6),(2.7). It is known from the analysis of Hartle and Hawking [19] that the propagator of the scalar particle in the Schwarzschild metric is an analytical function of U and V in the upper half-plane of the complex U-plane and in the lower half-plane of the complex V-plane. In terms of the variable z this means that the propagator remains an analytical function when it is continued from the real semi-axes $z > 0$ in the clockwise direction over the angle 2π. There is a slight distinction in our case. Our analysis relies on the wave function, while the work [19] refers to the properties of the propagator. However, the analytical properties of the wave function are similar to those of the propagator. We conclude that the analytical continuation used in derivation of (3.2) is justified. In contrast, an attempt to use the analytical continuation rotating z from the region $z > 0$ in the counter-clockwise direction should meet a difficulty. It really does, as demonstrated an attempt discussed above.

Let us summarize the main ideas used in derivation of (3.1) and (3.3). We verified firstly that in the vicinity of the event horizon the semiclassical approximation is valid, while the classical action possesses the logarithmic singularity (2.4). Then we analyzed consequences of this singularity for the wave function using its analytical continuation in the complex r-plane in the vicinity of the event horizon.

The mixing coefficient (3.3) has interesting properties. Firstly, it vanishes in the classical limit $\hbar \to 0$. Therefore it has no analog in the classical description, in this sense it is an *unexpected* result, making its physical consequences also unexpected. Moreover, even in the quantum picture one needs to make an effort to distill it. Remember that from the first sight the gravitational field is taken into account in the radial incoming and outgoing waves in (2.10) adequately and completely. However, the result obtained in (3.3) indicates that there exists some part of the gravitational interaction that remains unaccounted for in these wave functions, call it an "additional" interaction. Its existence can be suspected since the wave functions in (2.10) are singular on the horizon. The argument in favor of a possible additional interaction can also be drawn from the singular nature of the potential (2.14) in the Schrödinger-type equation (2.12). Dealing with the potential that is so singular one can suspect that some part of this potential, the δ-term localized on the horizon, i. e. proportional to $\delta(r-1)$ or its derivatives, may remain unaccounted for. This part, if exists, would mix the incoming and outgoing waves. The suspected additional singularity of the potential has to be moderated

by the causality conditions. This means that one can consider the adiabatic "switched off" of the black hole in the distant past. A precise mechanism for this switching off is not relevant, it is sufficient to assume only that this procedure is foreseeable. When the black hole is switching off, the singularity on the horizon disappears. That is why one can use a method of the analytical continuation. The analytical conditions are closely related to the causality principle, incorporating its consequences. Developing this argument we take the wave function (3.1) close to the horizon, but outside it, $|r-1| \ll 1,\ r>1$. In this region the gravitational field that exists for $r \neq 1$ is included in each of the two waves on the right-hand side almost completely, except for the possible δ-term that can manifest itself only through the mixing of these two waves. Imposing the causality condition through the analytical continuation we prove that this mixing really takes place. In our derivation the distance from the horizon can be made arbitrary small, $|r-1| \to 0$. This indicates that the mixing of the incoming and outgoing waves in (3.1) originates from those events that are localized on the horizon $r=1$, in accord with the expectation that the effect is due to a δ-term that is missed in the potential (2.13).

The logarithmic singularity of the radial action plays a central role in the above derivation. It is instructive to compare this singularity with the behavior of the classical trajectory. Recalling the classical equations of motion $\partial_\varepsilon S(\mathbf{r},r)=0$ one observes that the well-known exponential function in the trajectory (2.5) and the logarithmic singularity of the action are simply one and the same property expressed by two different means. Thus the reflection property found is closely related to the exponential-type behavior of the classical trajectory.

The result given in (3.1),(3.3) indicates that the horizon is capable of reflecting the incoming particle with probability given in (1.1). By the same token it means that the probability for the incoming particle to cross the horizon $\mathcal{P}_{\rm cr}$ penetrating into the inside region is less than unity

$$\mathcal{P}_{\rm cr} = 1 - \mathcal{P} \ . \tag{3.6}$$

We will use (3.6) in Section V discussing the Hawking radiation process. Summarizing, it is demonstrated that the black hole is capable to reflect particles that come to its horizon, the reflection probability satisfies (1.1).

IV. REFLECTION BY DIFFERENT TYPES OF BLACK HOLES

This Section extends the results derived above for the Schwarzschild black hole to other, more complex types of black holes. We rely on the step-by-step approach considering first the Reissner-Nordström solution, then the Kerr solution and only after that the general Kerr-Newman solution. The reader familiar with these solutions may prefer to go directly to Section IV C that discusses the general case.

A. Charged black holes

Consider the Reissner-Nordström black hole with the mass m and charge q. Its metric is given by

$$ds^2 = -\left(1-\frac{1}{r}+\frac{q^2}{r^2}\right)dt^2 + \frac{dr^2}{1-1/r+q^2/r^2} + r^2 d\Omega^2 \ . \tag{4.1}$$

The Hamilton-Jacobi equation for the particle with the mass μ, charge Q and orbital momentum L for the metric (4.1) reads

$$\frac{\left(\dot S - Q\Phi\right)^2}{1-1/r+q^2/r^2} = \left(1-\frac{1}{r}+\frac{q^2}{r^2}\right)\left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 + \mu^2 \ , \tag{4.2}$$

where $\Phi(r) = q/r$ is the black hole electric potential (compare Eq.(2.2). Separating the variables $S(\mathbf{r},t) = -\varepsilon t + L\varepsilon + S(r)$ one derives

$$S(r) \ = \ \mp \int \left[(\varepsilon - Q\Phi(r))^2 - \left(\mu^2 + \frac{L^2}{r}\right)\left(1-\frac{1}{r}+\frac{q^2}{r^2}\right)\right]^{1/2} \times \frac{dr^2}{1-1/r+q^2/r^2} \ . \tag{4.3}$$

The poles of $g_{rr} = 1-1/r+q^2/r^2$ are located on two spherical surfaces with radiuses

$$r_\pm = \frac{1}{2} \pm \sqrt{\frac{1}{4}-q^2} \ . \tag{4.4}$$

The largest of them with the radius r_+ represents the black hole horizon. In the vicinity of the horizon $r \to r_+$ one finds from (4.3)

$$S(r) \simeq \mp \zeta \, \ln(r-r_+) \ , \tag{4.5}$$

where

$$\zeta = [\,\varepsilon - Q\Phi(r_+)\,]\frac{r_+^2}{r_+ - r_-} \ . \tag{4.6}$$

In analogy with (2.4) the action (4.5) possesses the logarithmic singularity. We can therefore follow the way paved by Eqs.(3.1),(3.2) and (3.3). Firstly we construct the wave function

$$\phi(r) = \exp[-i\,\zeta \ln(r-1)\,] + \mathcal{R}\exp[\,i\,\zeta\ln(r-1)\,], \tag{4.7}$$

that describes the radial motion of the particle in the vicinity of the event horizon. Then, introducing the variable $z = r - r_+$, and assuming that $z>0$, $|z| \ll r_+ - r_-$, i. e. taking r in the external region in a close vicinity of

the event horizon, we make the analytical continuation rotating z in the complex plane $z \to \exp(-i\gamma)z$, $\gamma \geq 0$, eventually taking $\gamma = 2\pi$. This procedure gives the coefficient of reflection $\mathcal{R} = \exp(-2\pi\zeta)$ and the probability of reflection $\mathcal{P} = |\mathcal{R}|^2 = \exp(-4\pi\zeta)$. The latter result agrees with (1.1), where the value for the parameter T follows from (4.5),(4.6)

$$kT = \frac{\hbar c}{4\pi} \, \frac{r_+ - r_-}{r_+^2} \ , \tag{4.8}$$

(absolute units). It proves equal to the Hawking temperature of the charged black hole.

B. Rotating black holes

Consider the Kerr black hole that possesses the mass m and the spin j that is conveniently parameterized by $a = j/m$. The Kerr metric in the Boyer-Lindquist coordinates reads

$$\begin{aligned} ds^2 = \; & - \; \frac{\Delta}{\rho^2}(\, dt - a \sin^2\theta \, d\varphi \,)^2 \\ & + \; \frac{\sin^2\theta}{\rho^2} \Big[\, (r^2 + a^2) \, d\phi - a \, dt \,\Big]^2 \\ & + \; \frac{\rho^2}{\Delta} \, dr^2 + \rho^2 \, d\theta^2 \, . \end{aligned} \tag{4.9}$$

Here $\Delta = r^2 - r + a^2$ and $\rho^2 = r^2 + a^2 \cos^2\theta$. The nodes of Δ, are located on the two spheres with radiuses

$$r_\pm = \frac{1}{2} \pm \sqrt{\frac{1}{4} - a^2} \ , \tag{4.10}$$

the largest of which represents the black hole horizon. The Hamilton-Jacobi equations of motion for the metric (4.9),

$$\begin{aligned} & \frac{1}{\Delta\,\rho^2} \left[(r^2 + a^2) \, \frac{\partial S}{\partial t} + a \, \frac{\partial S}{\partial \varphi} \right]^2 \\ - \; & \frac{1}{\rho^2 \sin^2\theta} \left[a \, \sin^2\theta \, \frac{\partial S}{\partial t} - \frac{\partial S}{\partial \varphi} \right]^2 \\ - \; & \frac{\Delta}{\rho^2} \left(\frac{\partial S}{\partial r} \right)^2 - \frac{1}{\rho^2} \left(\frac{\partial S}{\partial \theta} \right)^2 = \mu^2 \ , \end{aligned} \tag{4.11}$$

allow the full separation of variables $S(\mathbf{r}, t) = -\varepsilon t + J\varphi + \Sigma(\theta) + S(r)$ that produces the following result for the radial action $S(r)$

$$S(r) = \int \Delta^{-1} \sqrt{R} \; dt \ , \tag{4.12}$$

$$R = P^2 - \Delta \, [\, \mu^2 r^2 + K \,] \ , \tag{4.13}$$

$$P = \varepsilon \, (r^2 + a^2) - aJ \ , \tag{4.14}$$

Here J is the conserved projection of the orbital momentum of the particle on the axis of rotation of the black hole, and K is an additional ("accidental") integral of motion. In the vicinity of the horizon $r \to r_+$, $\Delta \to 0$, one finds from (4.12) that $S(r)$ has a logarithmic singularity that satisfies (4.5) in which the parameter ζ equals

$$\zeta = (\varepsilon - J\omega) \frac{r_+^2 + a^2}{r_+ - r_-} \ . \tag{4.15}$$

Here $\omega = a/(r_+^2 + a^2)$ is the frequency of rotation of the black hole horizon. Using the method well-discussed above we derive from the logarithmic singularity that the rotating black hole is capable of reflection, the probability of reflection is given by (1), in which (4.15) predicts for the parameter T

$$kT = \frac{\hbar c}{4\pi} \, \frac{r_+ - r_-}{r_+^2 + a^2} \ , \tag{4.16}$$

(absolute units) that coincides with the temperature of the rotating black hole.

C. Charged-rotating black holes

Consider the general case of the Kerr-Neumann black hole that possesses both the charge q and the spin j. The Kerr-Newman metric in the Boyer-Lindquist coordinates is described by Eq.(4.9) in which the parameter Δ reads

$$\Delta = r^2 - r + a^2 + q^2 \ . \tag{4.17}$$

The nodes of Δ are located on the spheres with radiuses

$$r_\pm = \frac{1}{2} \pm \sqrt{\frac{1}{4} - a^2 - q^2} \ , \tag{4.18}$$

the largest of which represents the black hole horizon. The electromagnetic field of the black hole is described by the vector potential $A_\mu dx^\mu = -(qr/\rho^2)(dt - a \sin^2\theta \, d\varphi))$. The Hamilton-Jakobi equations of motion for a charged particle moving in the gravitational and electromagnetic fields created by a black hole allow the full separation of variables, see e.g. p.901 of Ref.[15]. One promptly finds that the radial action is described by Eqs.(4.12),(4.13) in which the parameter P equals

$$P = \varepsilon \, (r^2 + a^2) - aJ - qQr \ . \tag{4.19}$$

From Eqs.(4.12),(4.13),(4.19) we find that on the horizon $r \to r_+$ the action has the logarithmic singularity

$$S(r) \simeq \mp \, \zeta \, \ln(r - r_+) \ , \tag{4.20}$$

where

$$\zeta = [\, \varepsilon - Q\Phi(r_+) - J\omega \,] \, \frac{r_+^2 + a^2}{r_+ - r_-} \ . \tag{4.21}$$

Here $\Phi(r_+) = qQr_+/(r_+^2 + a^2)$ is the potential describing interaction of the particle with the electromagnetic

field of the black hole on the horizon. Using Eq.(4.21) and applying the method well described above one proves that the reflection probability for the Kerr-Newman black hole is given by (1.1). The parameter T that appears in (1.1) satisfies Eq.(4.16) with $r_\pm$ from Eq.(4.18); this T coincides with the temperature of the Kerr-Newman black hole. Setting in Eqs.(4.16),(4.18) either q, or j, or both of them to zero one returns to the case of the Kerr black hole, the Reissner-Nordström black hole, and the Schwarzschild black hole respectively.

We relied above on the semiclassical approach. Eq.(4.18) can be improved to account more accurately for the quantum properties of the momentum j by substituting $j^2 \to j(j+1)$ in a^2 in (4.18). This issue becomes important when quantum properties of the black hole itself are considered, see recent works of Bekenstein devoted to this subject [20, 21]. However, for the purpose of this work this subtlety is not essential.

We discussed in this Section several types of black holes that possess either the charge, or momentum or both, verifying that in each and every case the black hole is capable of reflection. Our most general result, which is presented for the Kerr-Newman solution, is described in Eqs.(1.1),(4.16),(4.18). There are known a number of more sophisticated solutions for the black holes with hair, see the review [22], but we leave them outside the scope of the present work.

V. INTERFERENCE, REFLECTION AND RADIATION

Let us show that the reflection ability of the black hole and the phenomenon of Hawking radiation have the same physical origin, interference of the incoming and outgoing waves. Consider the Schwarzschild case for simplicity. It is convenient to rewrite the radial wave function (3.1) in a more formal abstract notation

$$|\,\phi\,\rangle = |\,\mathrm{in}\,\rangle + \mathcal{R}\,|\,\mathrm{out}\,\rangle\ . \tag{5.1}$$

This notation assumes that the time-dependent factor is included in the wave function, i. e. $|\,\phi\,\rangle = \exp(-i\varepsilon t)\phi(r)$, where $\phi(r)$ is given in (3.1). The two terms on the right-hand side of (3.1) give the corresponding terms in (5.1) that can be conveniently written using Kruskal coordinates (2.6),(2.7) as

$$|\,\mathrm{in}\,\rangle \;=\; \exp[-i\varepsilon\,\ln(V^2)\,]\ , \tag{5.2}$$
$$|\,\mathrm{out}\,\rangle \;=\; \exp[\;\, i\varepsilon\,\ln(U^2)\,]\ . \tag{5.3}$$

We restrict our discussion to the events that take place in the vicinity of the horizon where the semiclassical description holds, justifying (5.2),(5.3). The classical trajectory that corresponds to the incoming wave $|\,\mathrm{in}\,\rangle$ follows from the equation of motion $\partial_\varepsilon S = 0$, where the action reads $S = \varepsilon\,\ln(V^2)$. Therefore the ingoing trajectory is described by equation $V = const$. Similarly the outgoing wave $|\,\mathrm{out}\,\rangle$ in the vicinity of the horizon corresponds to the classical trajectory $U = const$. In r, t variables these two trajectories are presented in (2.5) for the outside region.

Fig. 1 shows classical trajectories in Kruskal coordinates. This graphical presentation emphasizes the unexpected, nontrivial nature of the interference between the incoming and outgoing waves in (5.1). A particle that follows the incoming trajectory has no classically allowed chance to switch to the outgoing trajectory in the classical approximation. Fig. 1 visualizes this argument, showing that inside the event horizon the incoming and outgoing trajectories belong to different regions of the $U-V$ plane. Thus the incoming and outgoing trajectories seem to be completely unrelated. However, equation (5.1) indicates that on the quantum level there arises a connection between the incoming and outgoing waves. It manifests itself as the interference of these waves in the wave function. We verified this statement above for the outside region $r > 1$, but it holds for the inside region as well. Indeed, Kruskal coordinates in Eqs.(5.2),(5.3) show that the logarithmic singularity of the wave function does not depend on the sign of U and V, i.e. it exists on both sides of the horizon. Therefore inside the horizon one

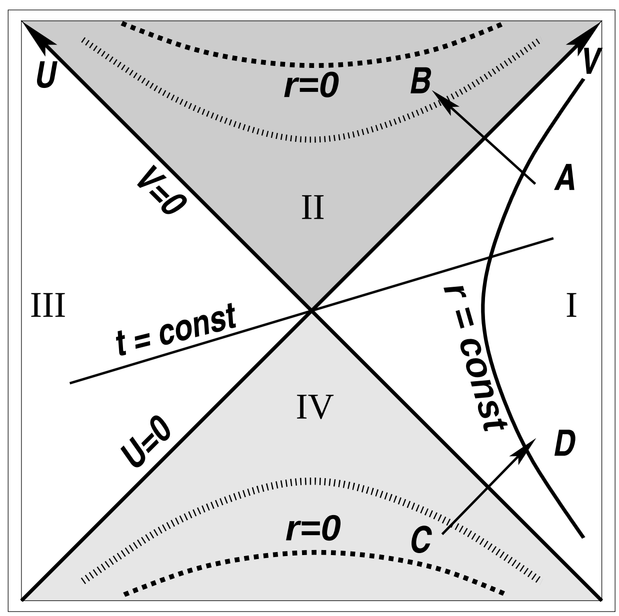

FIG. 1: *Kruskal coordinates. Areas I and III represent two identical copies of the outside region; II, IV show two inside regions. Hyperbolic curves $UV = const$ describe condition $r = const$, the dotted curve shows location of $r = 0$, the inclined straight line presents condition $t = const$. The direction of time flow in I and III is opposite. The incoming particle follows AB crossing the horizon $U = 0$ and residing in II. The outgoing particle CD escapes from IV crossing the horizon $V = 0$ and coming to I. Areas II and IV are not connected, which ensures classical confinement in II. The wave function (3.1) or (5.1) describe mixing of events that correspond to incoming and outgoing classical trajectories (AB and CD), resulting in phenomena of reflection and radiation.*

can use same method that we used above for the outside region, which leads to the same result that remains valid on both sides of the horizon: the incoming and outgoing waves do interfere in the wave function (5.1), see also discussion in Ref.[17].

We discussed in Sections III and IV the physical manifestation of this interference for the outside region, claiming that it leads to the reflection of the probing incoming particle from the event horizon. Let us now consider the physical manifestation of this interference for the region inside the horizon. The classical ingoing trajectory $V = const$ describes here the motion towards the black hole center, the outgoing $U = const$ trajectory describes the motion that eventually brings the particle from the inside region, over the horizon, into the outside region $r > 1$. If a particle follows the ingoing classical trajectory then, as mentioned above, there is no classical way for it to switch to the outgoing trajectory and escape into the outside region. However, (5.1) shows that the perception based on the classical picture is not completely correct. In the quantum wave function the proper ingoing wave $|\,\mathrm{in}\,\rangle$ is mixed with the proper outgoing wave $|\,\mathrm{out}\,\rangle$. This mixing indicates that the particle that moves towards the black hole center in the inside region has a finite chance to simultaneously populate the outgoing wave that brings it to the outside region. Thus there is a finite probability for the particle to escape from the region inside the horizon into the outside region.

Let us calculate this probability. Suppose that there is a particle confined in the inside region. Assume that this particle occupies a state with the quantum numbers ε, L, M moving from the horizon deeper inside the black hole, eventually aiming at the singularity at the origin [23]. According to the above discussion one should describe this particle by the wave function (5.1), which shows that there is an admixture of the outgoing wave. The probability to populate this wave is $\mathcal{P} = |\mathcal{R}|^2$. Following the classical outgoing trajectory, which corresponds to this wave, the particle can reach the event horizon and therefore can escape into the outside world.

Thus there exists the probability that the particle escapes $\mathcal{P}_{\rm esc} \propto |\mathcal{R}|^2 = \mathcal{P}$. We can be more specific. We know that the wave that reaches the event horizon is partially reflected. According to (3.6) the probability of reflection equals $\mathcal{P}_{\rm cr} = 1-\mathcal{P}$. We proved this result when we considered the scattering process that takes place in the outside region. One can verify that this result holds when we consider the scattering that takes place for the wave that comes to the horizon on its way from inside-out as well. Combining the two factors, the probability to populate the outgoing wave, and the probability to cross the event horizon we conclude that the probability for the particle to escape into the outside world equals $\mathcal{P}_{\rm esc} = \mathcal{P}(1-\mathcal{P})$. It is instructive to compare this result with the probability of the particle to be absorbed. Suppose we have an incoming particle in the outside region in a state described by the wave function (5.1) with the same quantum numbers ε, L, M. The probability for this particle to populate the ingoing wave in (5.1) is unity, therefore the probability to be absorbed $\mathcal{P}_{\rm abs}$ into the inside region equals the probability to cross the event horizon (3.6), which gives $\mathcal{P}_{\rm abs} = 1-\mathcal{P}$. We can consider now the ratio of the probability for a particle to escape from the inside region to the probability to be absorbed

$$\frac{\mathcal{P}_{\rm esc}}{\mathcal{P}_{\rm abs}} = \mathcal{P} = \exp\left(-\frac{\varepsilon}{kT}\right)~. \tag{5.4}$$

Discussing the probabilities above we considered only those factors that originate directly from the wave function (5.1). The physical probabilities include also additional normalization factors related to the flux of particles and the surface area of the event horizon. However, these additional factors are canceled out in the ratio (5.4), which presents therefore the result for the ratio of the two physical rates, emittance and absorption. It states that the ratio of the emittance and absorption rates coincides with the conventional temperature factor that describes the ratio of these rates for the black body with the temperature T. This means that if the black hole is put inside the thermostat with the temperature T, then it remains in equilibrium with it. One concludes therefore that (5.4) indicates that the black hole possesses the temperature T radiating as a black body with this temperature, as was first discovered by Hawking [8, 9] using different arguments.

There is a conventional physical explanation for the Hawking process that refers to the creation of pairs. The gravitational field in the vicinity of the horizon creates a pair, then a particle goes into the outside world, while its anti-partner is absorbed by the black hole. This explanation of the process needs endeavor to approve the fact that the antiparticle brings into the black hole the negative amount of energy that compensates the energy of the created particle. Eq.(5.1) suggests an alternative simple explanation. The radiation happens because the particle confined inside the horizon can escape into the outside world. This point of view has automatically accounts for the reduction of the mass of the black hole; when the particle escapes from the black hole it contributes to the mass of the black hole no more.

Summarizing, we verified that both the reflection and the Hawking radiation stem from the interference of the incoming and outgoing waves in the wave function (5.1).

VI. DISCUSSION AND CONCLUSION

The existence of the event horizon that separates the outside and inside regions is the main property of black holes. It is well known that one can always choose the coordinate frame that makes the metric smooth on the horizon. Correspondingly, the classical equations of motion for a probing particle in these coordinates are also smooth on the horizon. From this fact follows a known conclusion: a probing particle that follows the classical trajectory on its way to the black hole crosses the horizon

quite smoothly, but after that will be forced to stay inside forever. However, quantum corrections influence the fate of this particle. Presented arguments indicate that the horizon makes a strong impact on the wave function of a probing particle. It manifests itself in the form of interference, mixing of the incoming and outgoing waves in the wave function (5.1). Without this mixing the incoming wave crosses the event horizon quite uneventfully, in accord with similar smooth transition through the horizon of the classical trajectory. The mentioned mixing indicates that the incoming wave inevitably incorporates some admixture of the outgoing wave with interesting consequences, that are discussed below. But first let us recall some details related to the interference *per se*. We verified the existence of the mixing, showed that it happens due to events localized on the horizon and calculated its magnitude using the semiclassical approximation. The central role played the classical action for a probing particle that possesses the logarithmic singularity on the horizon.

Importantly, this singularity persists in any coordinate frame, it exists even in those coordinates in which the metric is smooth on the horizon, for example, in Kruskal coordinates for the Schwarzschild metric. Using the analytical continuation of the semiclassical wave function in the vicinity of the horizon we found the coefficient $\mathcal{R}$ that describes the mixing of the incoming and outgoing waves in the wave function (5.1). This coefficient possesses a typically semiclassical nature for a classically forbidden quantity,

$$|\mathcal{R}| = \exp\left(-\frac{\mathcal{A}}{\hbar}\right) \ , \tag{6.1}$$

where A has the meaning of some effective classical action. For example, for the Schwarzschild geometry of the black hole $\mathcal{A} = \varepsilon\,\tau$, where τ has the dimension of time with the typical value $\tau = 2\pi r_g/c$. In the classical limit $\hbar \to 0$ the mixing (6.1) disappears. Thus, from the point of view of the classical approximation the physical manifestations of quantum interference look unusual. Having said that, it is necessary to point out that in more common scattering situations there is nothing unusual about the interference between the incoming and outgoing waves, on the contrary, it is quite normal. The point is that black holes are very special. They are supposed to absorb very well everything incoming, therefore naively there should exist only the incoming wave that describes the particle that approaches the horizon. From this perspective the existence of the interference and, consequently, existence of the reflected outgoing wave is surprising.

There were discussed two effects that originate from the interference between the incoming and outgoing waves. One of them is a novel effect, reflection. For any particle that approaches the event horizon there is a finite probability to bounce back, into the outside world. The probability of reflection depends on the energy ε of the incoming particle and the temperature T of the black hole. For $\varepsilon < T$ the black hole behaves as a reflector, which is unusual.

Another effect that follows from the interference of the incoming and outgoing waves is the well known phenomenon of the Hawking radiation. The suggested new explanation for this effect is simple and appealing. The radiation happens because when the incoming particle is confined in the inside region, it still maintains an opportunity to escape back into the outside world. This fact changes the perception of the event horizon. Conventional arguments claim that when the incoming particle comes into the inside region, it stays there forever, the horizon is impassable for the backward transition. This argument, however, holds only in the classical approximation. Quantum corrections make the horizon partially transparent, the particles can cross it and go away creating the Hawking radiation spectrum of the black hole.

Both the radiative and reflective abilities of black holes arise from quantum corrections, both these processes are governed by the Hawking temperature of the black hole, but experimentally they are well distinguishable. The reflected flux depends on the nature, flux and spectrum of incoming particles, as well as on the black hole properties, while the radiation is governed entirely by the black hole. The radiation phenomenon provides support for important thermodynamics properties of black holes. The suggested new approach to the origins of the radiation may help to look anew at the thermodynamics properties of black holes as well, but this topic lies ahead.

In conclusion, black holes are capable of reflection, this effect has a common physical origin with the Hawking radiation.

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[23] Generally speaking, to be certain that the particle is located in the inside region one needs to describe its motion with the help of the wave packet that propagates from the horizon into the deeper region. However, for our purposes it suffices to take into account only one wave with the given quantum numbers ε, L, M. Proving below that each wave of the packet has a chance to escape from the inside region, we prove simultaneously that the wave packet can escape as well.